



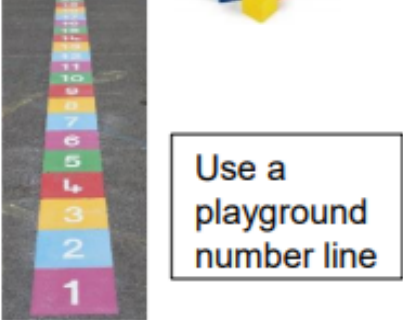

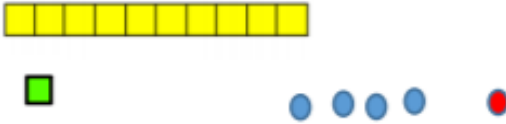
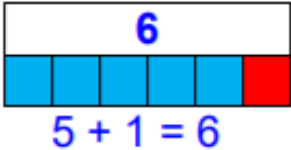
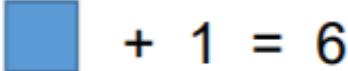
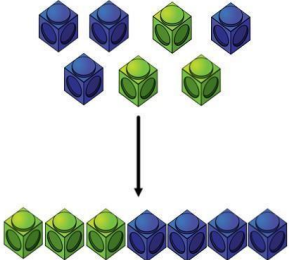
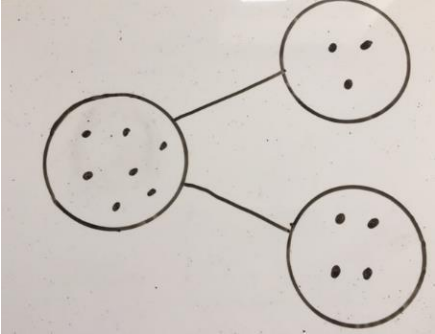
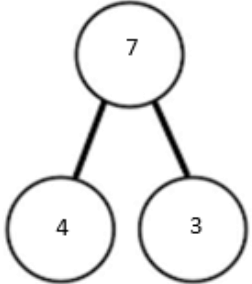
Shenfield St Mary's Primary School

Calculation Policy

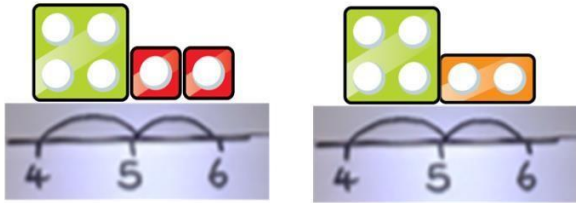
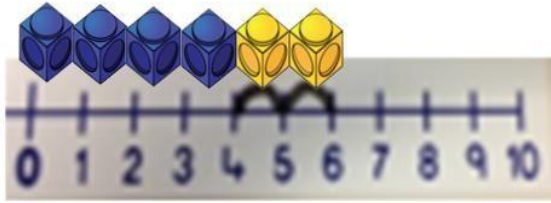
September 2018

Addition

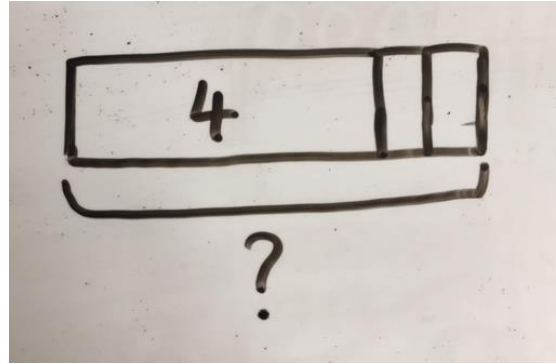
Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'

Concrete	Pictorial	Abstract
<p>Adding 1 more than...</p>  	<p>Use pictures of dienes or sticks and dots</p>  <p>Use bar model</p> 	$5 + 1 = 6$  $6 = \square + 1$
<p>Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars).</p> 	<p>Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.</p> 	<p>$4 + 3 = 7$ Four is a part, 3 is a part and the whole is seven.</p> 

Counting on using number lines using cubes or Numicon.



A bar model which encourages the children to count on, rather than count all.



The abstract number line:

What is 2 more than 4?

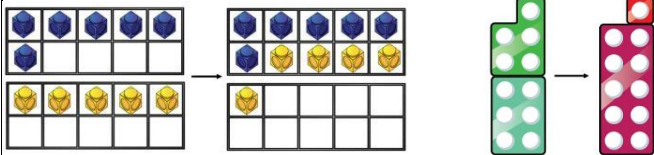
What is the sum of 2 and 4?

What is the total of 4 and 2?

$$4 + 2$$

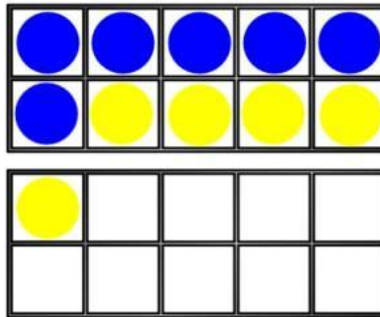


Regrouping to make 10; using ten frames and counters/cubes or using Numicon.



$$6 + 5$$

Children to draw the ten frame and counters/cubes.



Children to develop an understanding of equality e.g.

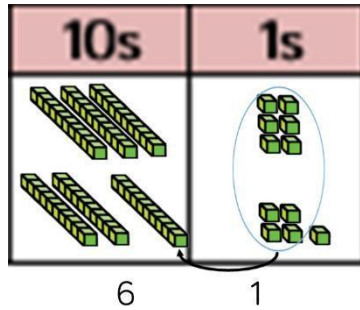
$$6 + \square = 11$$

$$6 + 5 = 5 + \square$$

$$6 + 5 = \square + 4$$

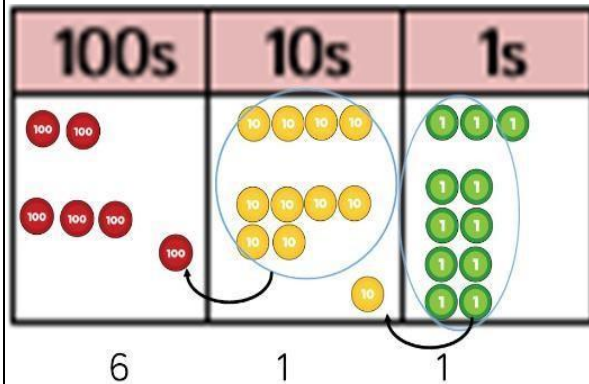
TO + TO using base 10. Continue to develop understanding of partitioning and place value.

$36 + 25$

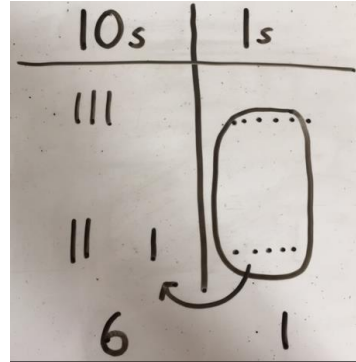


Add up the ones and exchange 10 ones for 1 ten.

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.



Children to represent the base 10 in a place value chart.



Looking for ways to make 10.

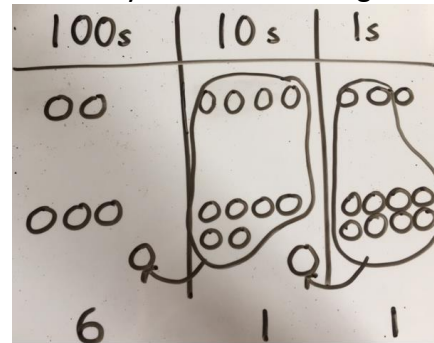
$$36 + 25 =$$

30 + 20 = 50
5 + 5 = 10
50 + 10 + 1 = 61

Formal method:

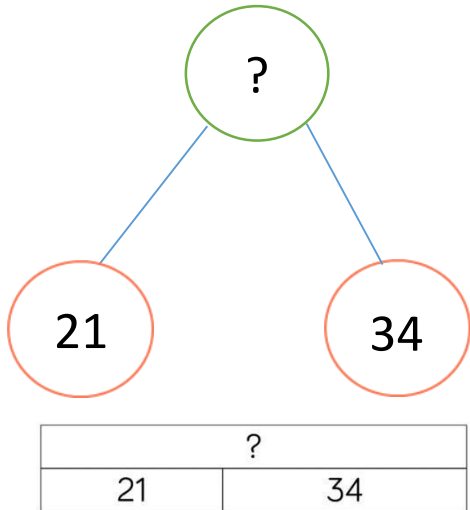
$$\begin{array}{r} +25 \\ 36 \\ \hline 61 \\ 1 \end{array}$$

Children to represent the counters in a place value chart, circling when they make an exchange.



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ 11 \end{array}$$

Conceptual variation; different ways to ask children to solve $21 + 34$



Word problems:

In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?

$21 + 34 = 55$. Prove it

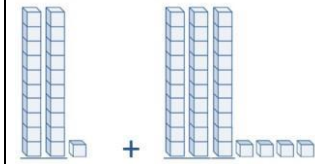
$$\begin{array}{r} 20 + 1 \\ 30 + 4 \\ \hline 50 + 5 = 55 \end{array}$$

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

$21 + 34 =$

= $21 + 34$

Calculate the sum of twenty-one and thirty-four.



Missing digit problems:

10s	1s
	?
?	5

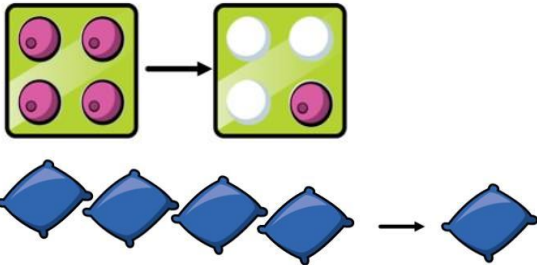
Subtraction

Key language: take away, less than, the difference, subtract, minus, fewer, decrease

Concrete

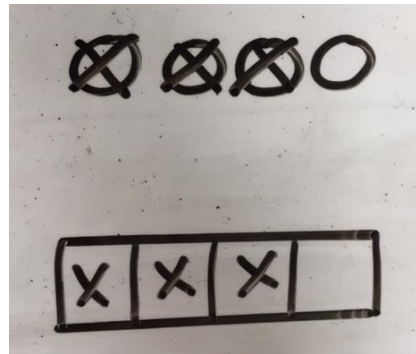
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used).

$$4 - 3 = 1$$



Pictorial

Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.

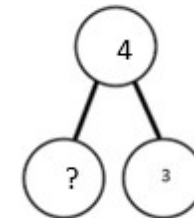


Abstract

$$4 - 3 =$$

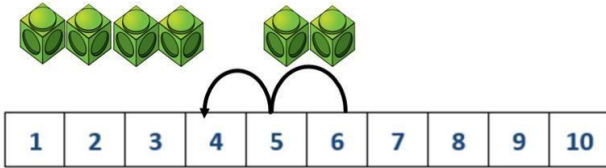
$$\square = 4 - 3$$

4	
3	?



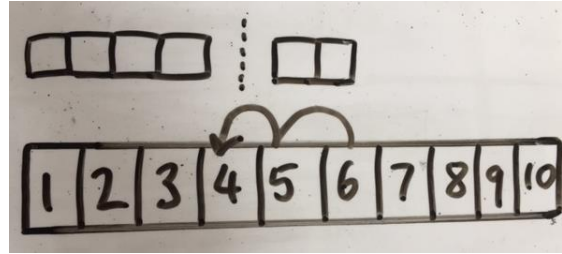
Counting back (using number lines or number tracks) children start with 6 and count back 2.

$$6 - 2 = 4$$

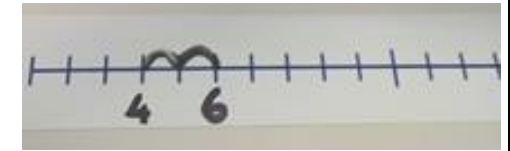


N.B. Alternatively children could count on from the smaller number.

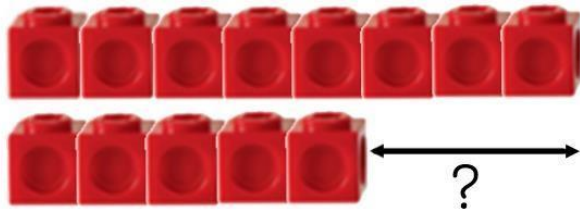
Children to represent what they see pictorially e.g.



Children put 6 in their head and count back 2. What number are they now at? Use their fingers to help.

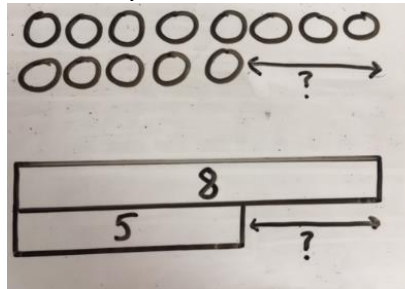


Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).



Calculate the difference between 8 and 5.

Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.



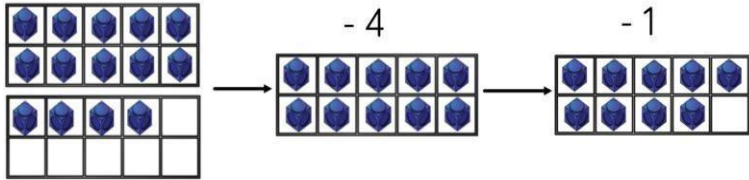
Find the difference between 8 and 5.

8 - 5, the difference is

Children to explore why $8 - 6 = 8 - 5 = 7 - 4$ have the same difference.

Making 10 using ten frames.

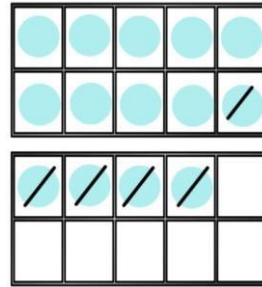
$14 - 5$



Make 14 on the ten frame. Take away the 4 first to make 10 and then take away 1 more so you have taken away 5 in total. You are left with the answer of 9.

Children to present the ten frame pictorially and

discuss what they did to make 10.



Children to show how they can make 10 by partitioning the subtrahend.

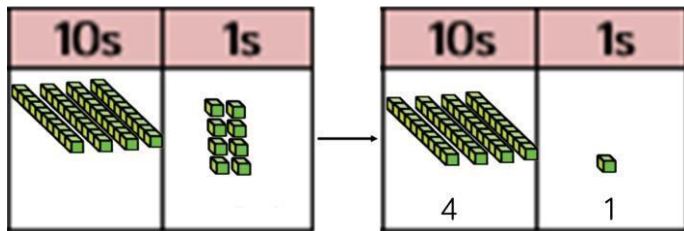
$$14 - 5 = 9$$

$$\begin{array}{c} 4 \quad 1 \end{array}$$

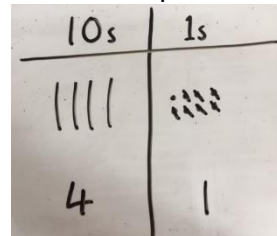
$14 - 4 = 10$
 $10 - 1 = 9$

Column method using base 10.

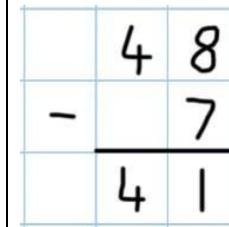
$48 - 7$



Children to represent the base 10 pictorially.



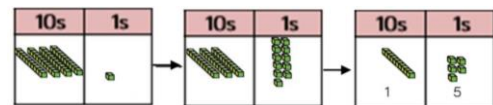
Column method or children could count back 7.



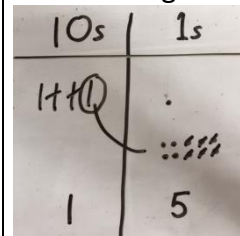
Column method using base 10 and having to exchange.

$41 - 26 =$

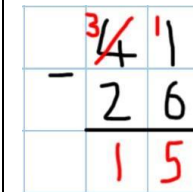
Start with 41 (4 tens + 1 one). Exchange one of the tens because you can't subtract 6 from the ones column as things stand. This leaves you with 3 tens and 11 ones, now you can subtract the 26 (2 tens and 6 ones) easily. Your final answer is 15.



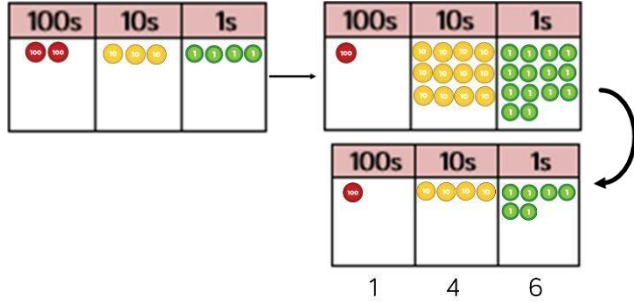
Represent the base 10 pictorially, remembering to show the exchange.



Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because $41 = 30 + 11$.



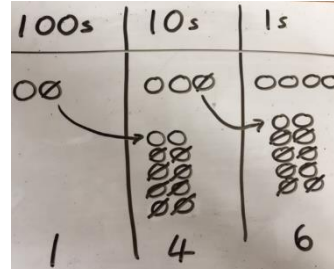
Column method using place value counters.



$$234 - 88$$

Again this relies on exchanging.

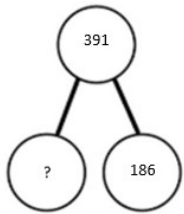
Represent the place value counters pictorially; remembering to show what has been exchanged.



Formal column method. Children must understand what has happened when they have crossed out digits.

$$\begin{array}{r} ^2 ^1 \\ 234 \\ - 88 \\ \hline 6 \\ \hline \end{array}$$

Conceptual variation; different ways to ask children to solve 391 - 186



391	
186	?

Raj spent £391, Timmy spent £186.
How much more did Raj spend?

Calculate the difference between 391 and 186.

$$\square = 391 - 186$$

$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

What is 186 less than 391?

Missing digit calculations

$$\begin{array}{r} ^3 ^9 ^{\square} \\ - ^{\square} ^{\square} 6 \\ \hline ^{\square} ^0 ^5 \end{array}$$

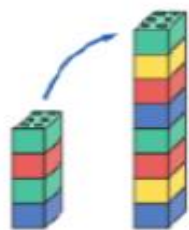
Multiplication

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups

Concrete

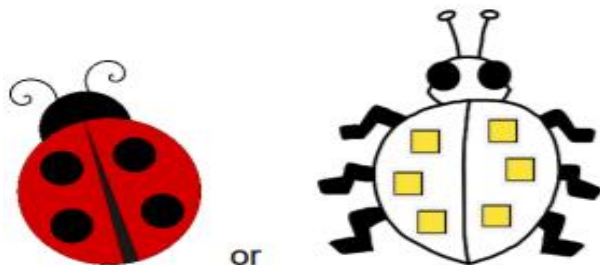
Doubling

Use practical activities to show how to double a number.



double 4 is 8
 $4 \times 2 = 8$

Use of dienes



or

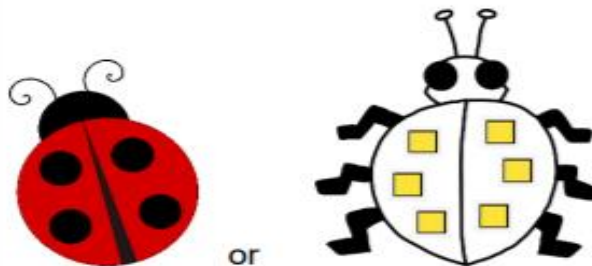
Pictorial

Draw pictures to show how to double a number.

Double 4 is 8

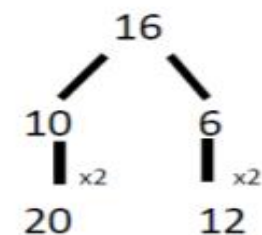


Diagrams to add dots or sticks to.



or

Abstract

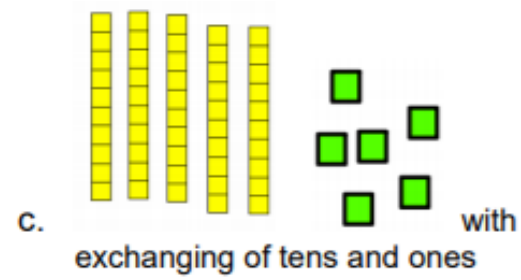
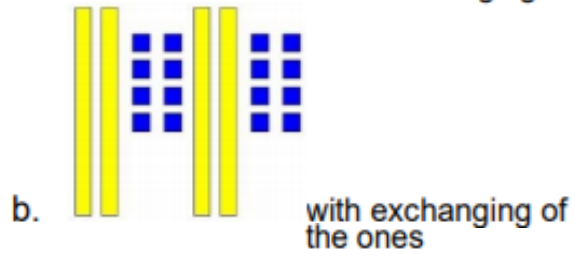
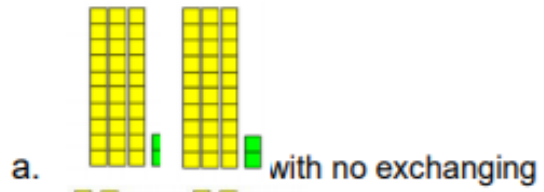


Partition a number and then double each part before recombining it back together.

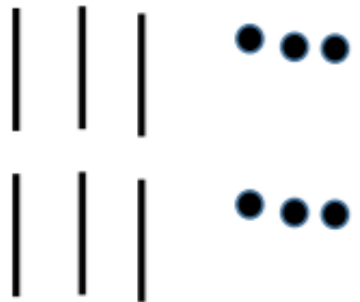
$$3 + 3 = 6$$

Doubling with double digit numbers

Use dienes



Use of sticks and dots to help with exchange process.



What is double 23?

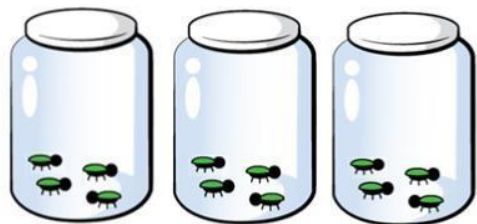
What is double 47?

What is double 87?

Repeated grouping/repeated addition

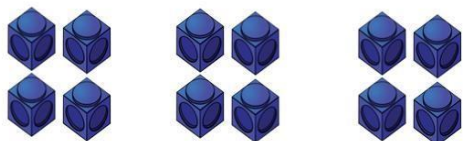
$$3 \times 4$$

$$4 + 4 + 4$$

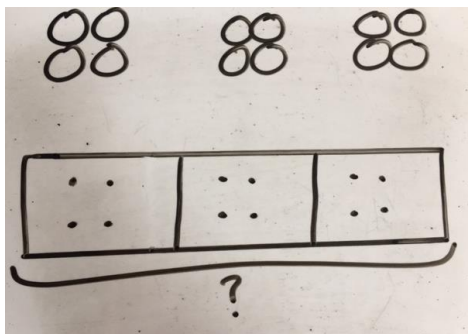


There are 3 equal

groups, with 4 in each group.



Children to represent the practical resources in a picture and use a bar model.

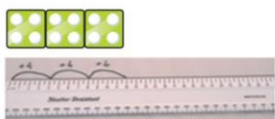


$$3 \times 4 = 12$$

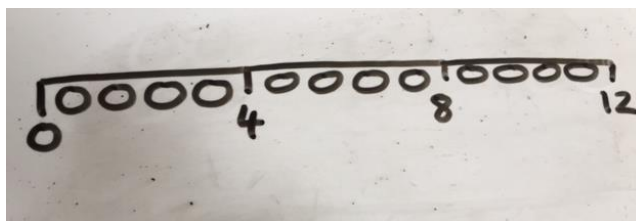
$$4 + 4 + 4 = 12$$

Number lines to show repeated groups 3×4

Cuisenaire rods can be used too

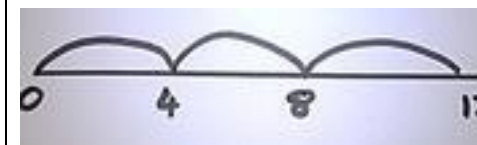


Represent this pictorially alongside a number line e.g.:



Abstract number line showing three jumps of four.

$$3 \times 4 = 12$$

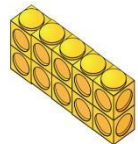


Use arrays to illustrate commutativity counters and other objects can also be used.

$$2 \times 5 = 5 \times 2$$

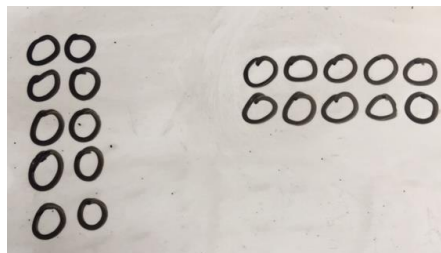


2 lots of 5



5 lots of 2

Children to represent the arrays pictorially.



Children to be able to use an array to write a range of calculations e.g.

$$10 = 2 \times 5$$

$$5 \times 2 = 10$$

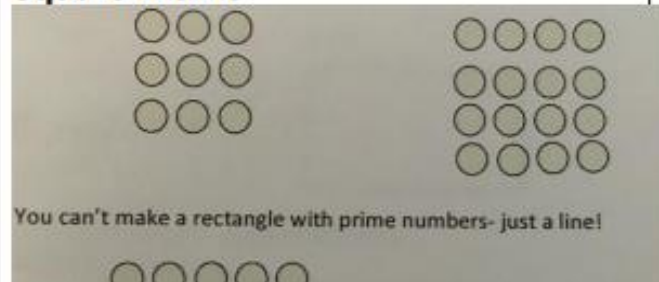
$$2 + 2 + 2 + 2 + 2 = 10$$

$$10 = 5 + 5$$

Finding out about rectangles

Use of dienes and arrays to investigate square numbers and prime numbers

'A square number is called a square number because you can make a square with it!'



Using arrays to investigate numbers

'Is 25 a square number?'

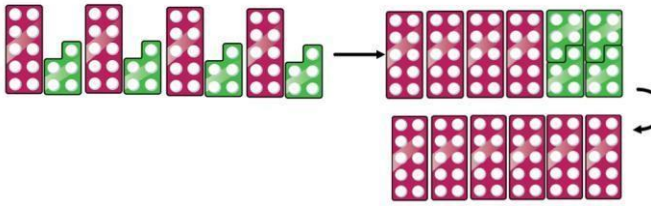
Why is 5 a prime number?



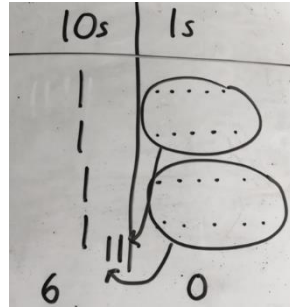
What is the square root of 25?

$$5^2 = ?$$

Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4×15



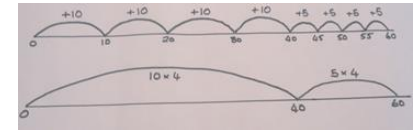
Children to represent the concrete manipulatives pictorially.



Children to be encouraged to show the steps they have taken.

$$\begin{array}{r} 4 \times 15 \\ \swarrow \searrow \\ 10 \quad 5 \end{array}$$

$$\begin{array}{l} 10 \times 4 = 40 \\ 5 \times 4 = 20 \\ 40 + 20 = 60 \end{array}$$

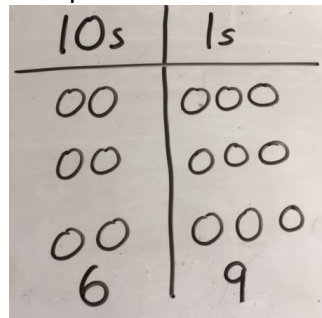


A number line can also be used

Formal column method with place value counters (base 10 can also be used.) 3×23

10s	1s
6	9

Children to represent the counters pictorially.



Children to record what it is they are doing to show understanding. 3×23

$$\begin{array}{l} 3 \times 20 = 60 \\ 3 \times 3 = 9 \\ 60 + 9 = 69 \end{array}$$

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

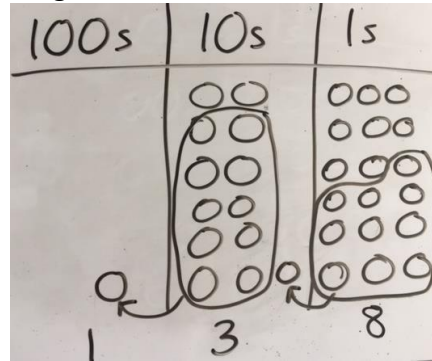
Formal column method with place value counters. 6×23

100s	10s	1s
	60	12



100s	10s	1s
1	3	8

Children to represent the counters/base 10, pictorially e.g. the image below.

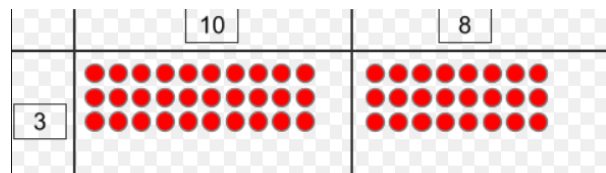


$$6 \times 23 =$$

Formal
written
method

$$\begin{array}{r} 23 \\ \times 6 \\ \hline 138 \\ \hline 11 \end{array}$$

Grid method for short multiplication



$$18 \times 3$$

First partition the 18 into 1 ten and 8 ones. Multiply each number by 3 and find the total.

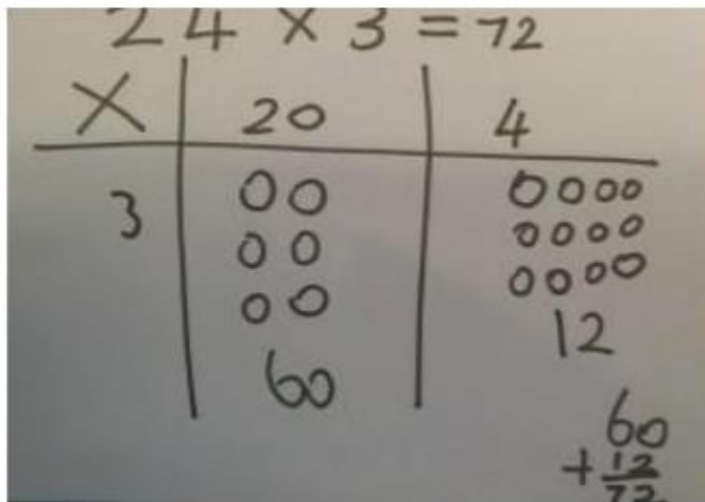
$$10 \times 3 = 30$$

$$8 \times 3 = 24$$

$$30 + 24 = 54$$

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

X	30	5
7	210	35

$$210 + 35 = 245$$

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

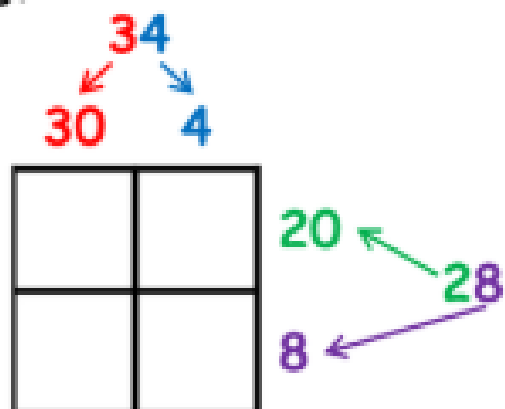
	10	8
10	100	80
3	30	24

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

This same process can be applied to multiplication as demonstrated below.

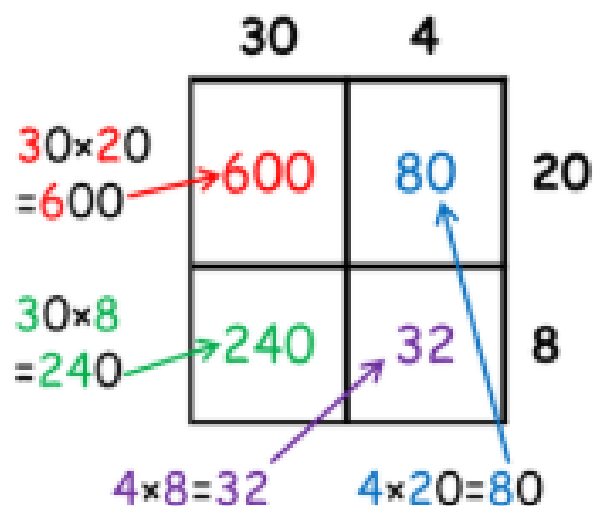
Step 1

To work out 34×28 ...
split each number into
tens and units, and
write next to a grid like
this:



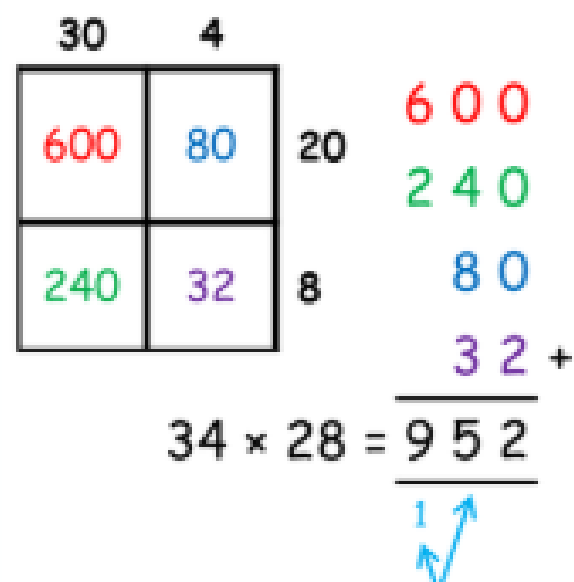
Step 2

Use times tables to fill
in the grid. Take care
with the number of
zeros needed!



Step 3

Add up the numbers
inside the grid.

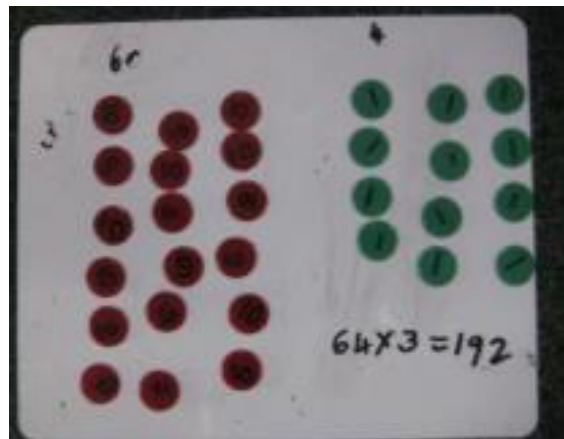


$8 + 3 + 4 = 15$
Carry the ten into
the next column.
Don't forget to
add it on!

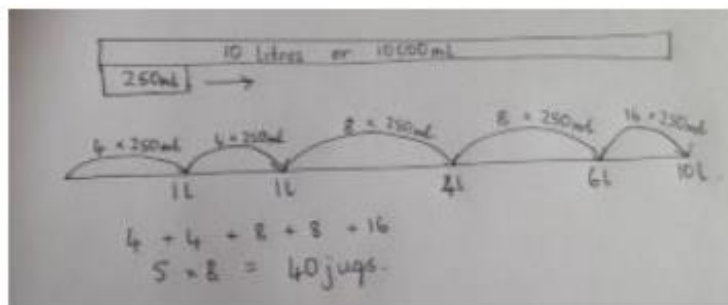
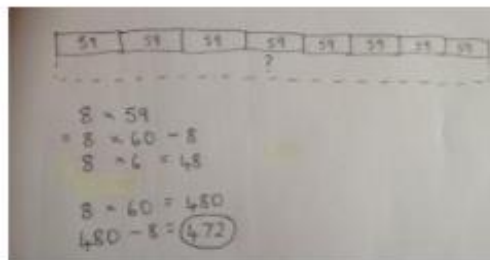
Column multiplication

Children can continue to be supported by place value counters at the stage of multiplication.

It is important that they always multiply the ones first and note down their answer followed by the tens which they note below.



Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



$$\begin{array}{r}
 1 \ 2 \ 4 \\
 \times \quad 2 \ 6 \\
 \hline
 7 \ 4 \ 4 \\
 1 2 \\
 2 \ 4 \ 8 \ 0 \\
 \hline
 3 \ 2 \ 2 \ 4 \\
 \hline
 1 \ 1
 \end{array}$$

Answer: 3224

Conceptual variation; different ways to ask children to solve 6×23

23	23	23	23	23	23
----	----	----	----	----	----

?

Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that $6 \times 23 = 138$

Find the product of 6 and 23

$$6 \times 23 =$$

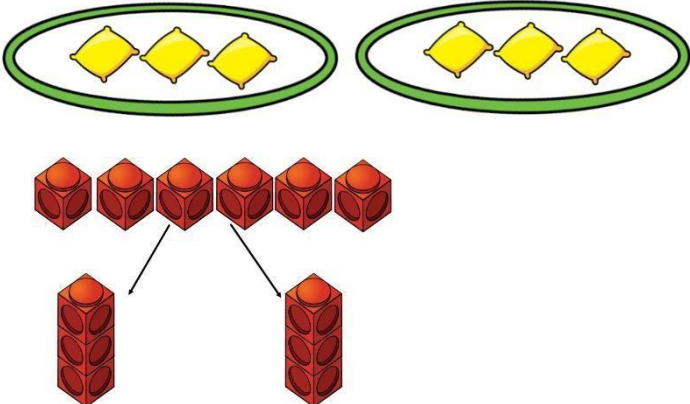
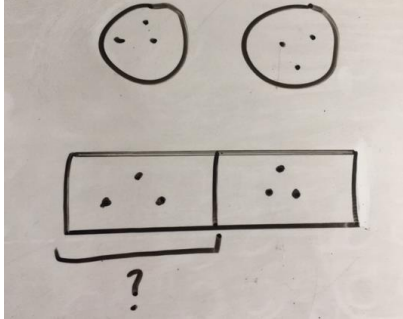
$$\begin{array}{r}
 \square = 6 \times 23 \\
 6 \quad 23 \\
 \times 23 \quad \times 6 \\
 \hline
 \hline
 \end{array}$$

What is the calculation? What is the product?

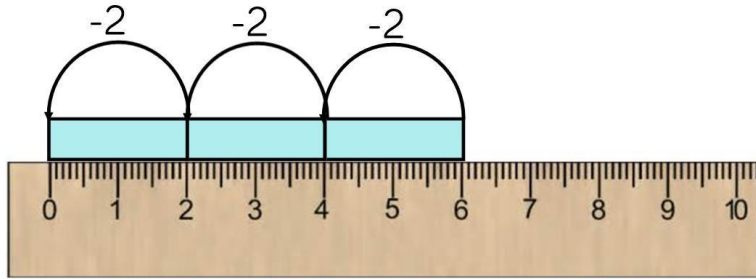
100s	10s	1s

Calculation Policy: Division

Key language: share, group, divide, divided by, half

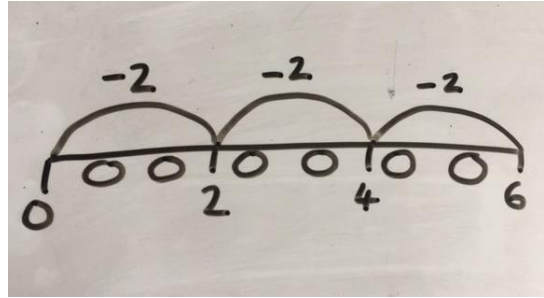
Concrete	Pictorial	Abstract		
<p>Sharing using a range of objects. $6 \div 2$</p> 	<p>Represent the sharing pictorially.</p> 	<p>$6 \div 2 = 3$</p> <table border="1" data-bbox="1554 472 2009 539"><tr><td>3</td><td>3</td></tr></table> <p>Children should also be encouraged to use their 2 times tables facts.</p>	3	3
3	3			

Repeated subtraction using Cuisenaire rods above a ruler. $6 \div 2$

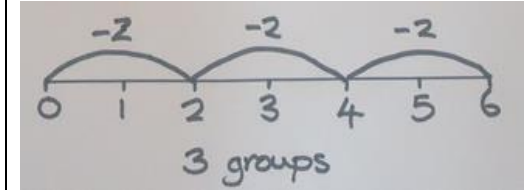


3 groups of 2

Children to represent repeated subtraction pictorially.



Abstract number line to represent the equal groups that have been subtracted.



Division within arrays

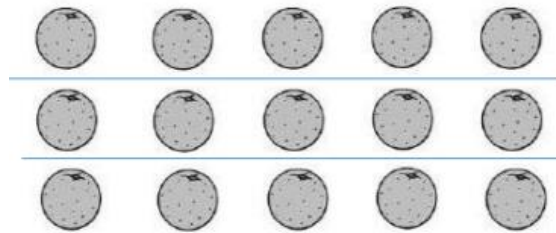


Link division to multiplication by creating an array and thinking about the

number sentences that can be created.

Eg $15 \div 3 = 5$ $5 \times 3 = 15$
 $15 \div 5 = 3$ $3 \times 5 = 15$

Use of
dienes



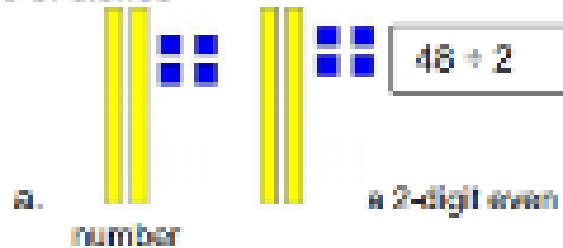
Draw an array and use lines to split the array into groups to make multiplication and division sentences.

Find the inverse of multiplication and division sentences by creating four linking number sentences.

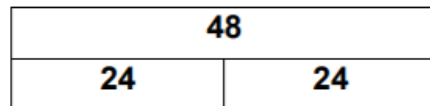
$7 \times 4 = 28$
 $4 \times 7 = 28$
 $28 \div 7 = 4$
 $28 \div 4 = 7$

Halving

Use of dienes



Use of bar model to represent halves



What is half of ...?

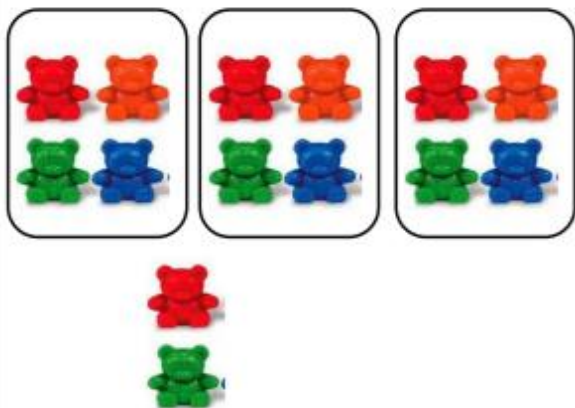
$\frac{1}{2}$ of ... = ?

$\frac{1}{2} \times 48 = ?$

Division with a remainder

$14 \div 3 =$

Divide objects between groups and see how much is left over



Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



Draw dots and group them to divide an amount and clearly show a remainder.



Complete written divisions and show the remainder using r.

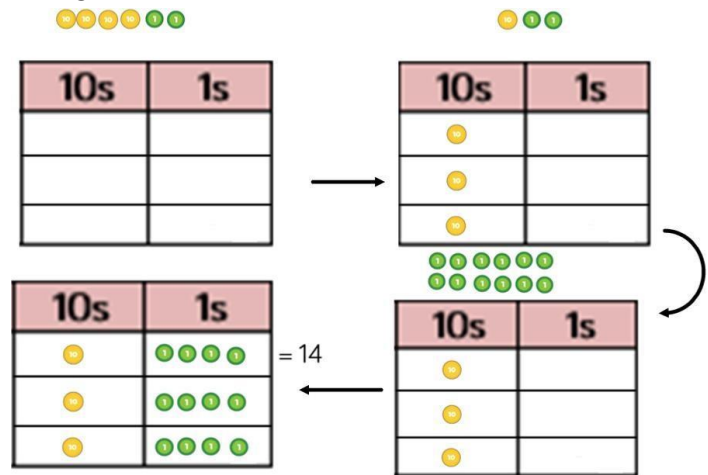
$29 \div 8 = 3 \text{ REMAINDER } 5$

↑ ↑ ↑ ↑

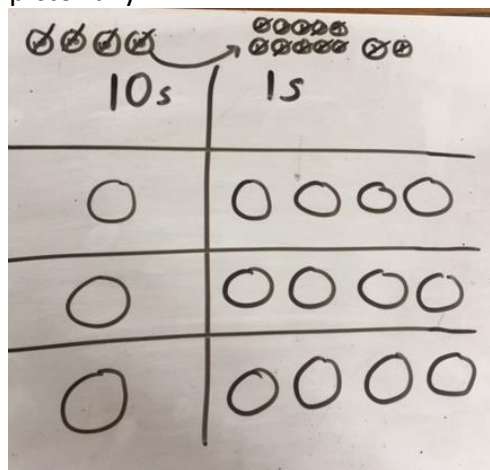
dividend divisor quotient remainder

Sharing using place value counters.

$$42 \div 3 = 14$$



Children to represent the place value counters pictorially.



Children to be able to make sense of the place value counters and write calculations to show the process.

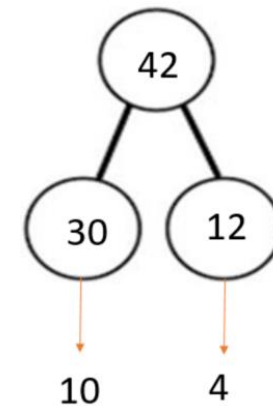
$$42 \div 3$$

$$42 = 30 + 12$$

$$30 \div 3 = 10$$

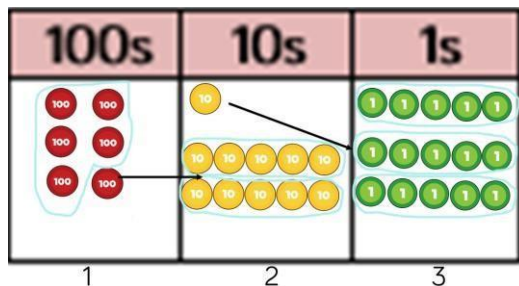
$$12 \div 3 = 4$$

$$10 + 4 = 14$$



Short division using place value counters to group.

$$615 \div 5$$



Make 615 with place value counters.

How many groups of 5 hundreds can you make with 6 hundred counters?

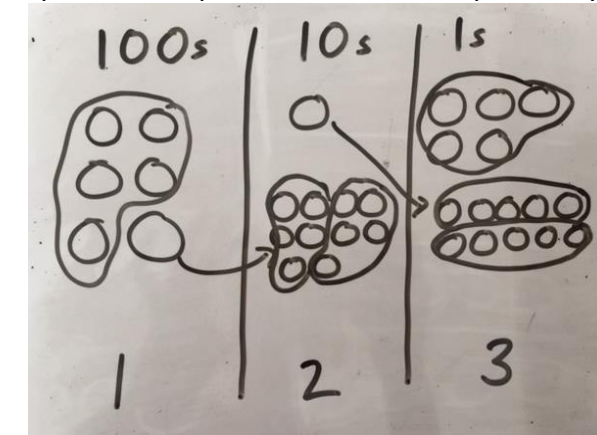
Exchange 1 hundred for 10 tens.

How many groups of 5 tens can you make with 11 ten counters?

Exchange 1 ten for 10 ones.

How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.



Children to the calculation using the short division scaffold.

$$5 \overline{) 615}$$

$$\begin{array}{r} 123 \\ 5 \overline{) 615} \\ \underline{5} \\ 11 \\ \underline{10} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

Chunking for short and long division

We show the children this method to demonstrate sharing and left over amounts. However, we later encourage the children to use the more standard method of division.

Short Division

$$196 \div 6 = 32 \text{ r } 4$$

	H	T	U	
6)	1	9	6	
	<u>6</u>	0		- (10 × 6)
	1	3	6	
	<u>6</u>	0		- (10 × 6)
		7	6	
	<u>6</u>	0		- (10 × 6)
		1	6	
	<u>1</u>	2		- (2 × 6)
			4	←

As children begin to use their knowledge of multiplication tables, particularly with multiples of ten for each table, they can progress from taking 10 chunks of a divisor to $30 \times 6 = 180$ and subtract this larger chunk.

$$10 + 10 + 10 + 2 = 32$$

4 remaining

Long Division

		2	3	r	4	
		<hr/>				
24		5	5	6		
		<hr/>				
		-4	8	0		24 × 20
		<hr/>				
			7	6		
		<hr/>				
		-7	2			24 × 3
		<hr/>				
				4		

Standard Long Division

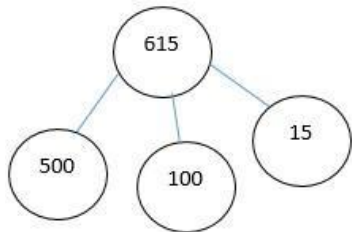
Encourage the children to write down the first 6 multiples of 48. 48 doesn't go into 7 but it goes into 76 once. Write 1 above the 6 in the answer space. Then $1 \times 48 = 48$ and $76 - 48 = 28$. Bring down the 2. 48 goes into 282 5 times. Write 5 above the 2 in the answer space. Then $5 \times 48 = 240$ and $282 - 240 = 42$. Bring down the 3. 48 goes into 423 eight times. Write 8 above the 3 in the answer space. Then $8 \times 48 = 384$ and $423 - 384 = 39$. Write remainder (R) 39 next to 158.

$$\begin{array}{r}
 158 \text{ R } 39 \\
 48 \overline{) 7623} \\
 \underline{-48} \\
 282 \\
 \underline{-240} \\
 423 \\
 \underline{-384} \\
 39
 \end{array}$$

48
96
144
192
240
288

Conceptual variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{) 615}$$

$$615 \div 5 =$$

$$\square = 615 \div 5$$

What is the calculation? What is the answer?

